Synopsis of proposed research work entitled

Mathematical Modeling of Some Mixed Boundary Value Problems of Thermoelasticity Using Fractional Differential Equations and Computational Software's

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Ph. D. SYNOPSIS

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Abstract of proposed research work:

1. Introduction:

Thermoelasticity describes the behavior of elastic bodies under the influence of nonuniform temperature fields. It represents, therefore a generalization of the theory of elasticity. The constitutive equations that is, the equations characterizing the particular material are temperature dependent and include an additional relation connecting the heat flux in the body with the local temperature gradient. This relation, known in its simplest form as Fourier's law, determines the temperature distribution in body.

The theory of thermoelasticity deals with the effects of the thermal state of an elastic body on the stress distribution and also with the effects of the stress on the temperature.

In theory of thermoelasticity there are two types of problems, first one is *Direct Thermoelastic Problem* and second one is *Inverse Thermoelastic Problem*. The direct thermoelastic problem consists of temperature, heat transfer conditions are prescribed on the surface of the boundary and conditions at interior points are to be determined. An inverse thermoelastic problem consists of determination of temperature of the heating media, heat flux on the boundary surface of the solids, when the condition of the displacement and stress are known at some points of the solid under consideration.

<u>Origin of the Research Problem:</u>

The theory of thermoelasticity was found in 1838 by **Duhamel J.M.C.**, who derived the equation for the strain and elastic body having temperature gradients and **Neumann** later derived the same results. The theory was based on the elastic strain and the thermal and mechanical effects, the total strain being determined by superimposing correct if the body is in thermal and mechanical equilibrium. But, in principle it is restricted when there are non stationary thermal and mechanical effects, since it does not describe strictly the motion associated with the thermal state, nor does it include the dynamic effects of the strain, or the interaction between the strain and temperature distributions. The classical uncoupled theory of thermoelasticity predicts two phenomena not compatible with physical observations. First, the equation of heat conduction of this theory does not contain any elastic terms; second, the

heat equation is of a parabolic type, predicting infinite speeds of propagation for heat waves.

2. Review of Literature and Development in the subject (Previous work done in the related area):

In 1956, **M. A. Biot** [1] developed the coupled theory of thermoelasticity. In this theory, the equations of elasticity and of heat conduction are coupled which deals with the first defect of the uncoupled theory. In 1957, **N. W. Nowacki** [2] developed the theory of thermoelastic diffusion. In this theory, the coupled thermoelastic model is used. In 1967, **Lord and Shulman** [3] derived the theory of generalized thermoelasticity, which is also referred as extended thermoelasticity theory, by modifying the Fourier's law of heat conduction with the introduction of a thermal relaxation time parameter. In 1980, **Dhaliwal** and **Sherief** [9] extended this theory to include the anisotropic case. In 1973, **Roy Choudhuri** [7] discussed the quasi-static thermal deflection of a thin clamped circular plate due to ramp type heating of a concentric circular region of the upper face, with the lower surface is kept at zero temperature and circular edge thermally insulated. In 1989, **N. Noda, F. Ashida and T. Tsuji** [10] discussed an analytical method for an inverse problem of three-dimensional transient thermoelasticity in a transversely isotropic solid by applying the Laplace and Fourier transforms with newly designed potential function and the practically applicability of the methods in engineering problems of great importance was illustrated. In 1993, **F. Ashida, N. Noda and I. A. Okumura** [11] discussed a new solution technique for thermoelastic problems of transversely isotropic solids in cylindrical coordinates.

In 2001, **S. G. Ingle and K. C. Deshmukh** [12] studied an inverse quasi-static thermal deflection of a thin clamped circular plate due to ramp type heating of a concentric circular region of the outer curved surface. In 2002, **K. Kim, and N. Noda** [13] studied two-dimensional unsteady thermoelastic problems of the FGM infinite hollow cylinder and the deflection of a functionally graded plate under transient thermal loading. In 2004, **F. Ashida, S. Sakata and H. Nakata** [14] solved an inverse problem of transient thermoelasticity in a piezoceramic hollow cylinder of crystal class. In 2005, **M. N. Gaikwad** [16] solved the inverse problem of thermoelasticity in a thin isotropic circular plate by determining the unknown temperature gradient, temperature distribution and the thermal deflection on the edge of the circular plate. In 2006, **Y. Ootao and Y. Tanigawa** [19] analyzed the theoretical treatment of transient thermoelastic problem involving a functionally graded hollow cylinder due to uniform heat supply. In 2009, **K. Deshmukh, S. Warbhe and V. Kulkarni [**20] studied the non-homogeneous steady-state heat conduction problem in a thin circular plate and its thermal stresses. In 2010, **K. Parihar and S. Patil** [22] determined the thermoelastic problems of

thin circular and rectangular plates. In 2012, **K. R. Gaikwad et al.,** [28] have studied the non-homogeneous heat conduction problem and its thermal deflection due to internal heat generation in a thin hollow circular disk. In 2013**, K. R. Gaikwad** [29] analyzed the thermoelastic deformation of a thin hollow circular disk due to partially distributed heat supply.

Fractional calculus has been used successfully to modify many existing models of physical processes. The first application of fractional derivatives was given by **Abel**, who applied fractional calculus in the solution of an integral equation that arises in the formulation of the tautochrone problem. **Caputo** [4, 8] and **Caputo** and **Mainardi** [5, 6] used fractional derivatives and their results found good agreement with the empirical evidences for description of viscoelastic materials. Study on thermoelasticity with fractional calculus was initiated by **Povstenko** [17, 18]. A review of thermoelasticity with fractional order heat conduction equation was given by **Povstenko** [21]. **Povstenko** [23] investigated thermal stresses in an infinite body with a circular cylindrical hole using the heat conduction equation based on Caputo time-fractional derivative. Recently, **Sherief et al.,** [24] developed a new theory of thermoelasticity using fractional calculus and proved the uniqueness theorem. Many researchers [25, 27, 30, 31] have contributed to the theory of fractional order thermoelasticity in recent times.

In 2004, **Y. Z. Povstenko** [15] have been studied a quasi-static uncoupled theory of thermoelasticity based on the heat conduction equation with a time fractional derivative of order α. In 2007, **Y. Z. Povstenko** [ ] investigated the fundamental solutions to the Cauchy problems as well as to the source problem in the case of spherical symmetry and corresponding thermal stresses. In 2009, **Y. Z. Povstenko** [ ] studied the theory of thermoelasticity based on the space-time fractional heat conduction equation. In 2010, **H. H. Sherief et al.,** [24] developed fractional order theory of thermoelasticity. In 2011**, M. A. Ezzat et al.,** [26] derived a new theory of thermodiffusion in elastic solids using the methodology of fractional calculus. In 2015, **Abbas, A. Ibrahim** [33] studied the fractional order theory of thermoelastic diffusion problem for an infinite medium with cylindrical cavity. In 2016, **Abbas, A. Ibrahim** [34] discussed an eigenvalue approach to fractional order thermoelasticity for an infinite body with a spherical cavity. In 2016, **J. J. Tripathi et al.,** [35] determined a dynamic problem of fractional order thermoelasticity for a thick circular plate with finite wave speeds.

3. Objectives of Research /Proposed Hypothesis:

The main objectives of this study are:

1. To study in detail some thermoelastic problems.
2. To study of existing methods.
3. To formulate new thermoelastic problem on solid and study their thermoelasticity.
4. To obtain an alternative solution of the problems by applying new techniques and compare our results with already obtain.
5. To study applications related with engineering field.

4. Methodology to be adopted:

1. To survey and collect relevant research papers, review articles, books, journals etc. through internet and by visiting various libraries.
2. Acquisition of knowledge about current research in the topic by participating in conferences, workshops and seminars.
3. To study the references and find out new results.
4. To carry out numerical calculations with the help of computational software like Maple, MATLAB, Mathematica, Mathcad, etc.
5. To find out applications of obtained result.
6. To publish the work in reputed journal.

5. Importance of the study:

Interdisciplinary Relevance:

The issue has great importance because it is an interdisciplinary work related with the branch of science and technology like Physics, Mechanical Engineering, and Civil Engineering etc. The isotropic and anisotropic thermoelastic problems to be considered have important industrial applications.

International and National Status:

Lot of work has been carried out by many National and International researchers on thermoelasticity. The works of few amongst them are quoted in the direct and indirect problems. (**Refer Origin of the Research Problem**).

Significance of the study:

The direct thermoelastic problems have applications in the field of Aeronautics. The high velocity of modern aircrafts gives rise to aeronautic heating. In turn, this produces intense thermal stresses by lowering the elastic limit and reduces the strength of aircraft structure and spacecrafts. While the inverse thermoelastic problem is very important in view of its relevance to various industrial machines subjected to heating such as main shaft of lathe,

turbine, and roll of rolling mill. It is also useful to analyze the experimental data and measurement of aerodynamic heating. The study of such problems will tremendously enhance the speed of the development in the industry, which will culminate in the overall development of the country.

Fractional calculus have been used successfully to modify many existing models of physical processes e.g., viscoelasticity, chemistry, electronics, wave propagation and biology. The differential equation of fractional order has been the focus of many studies due to their frequent appearance in various applications in fluid mechanics, physics, biology, engineering, etc.

6. Proposed work plan:

- **Year-wise plan of work and targets to be achieved are as follows:**

1. First year:

i) To collect the references in relevant field.
ii) To study the references in relevant field.
iii) To attend the seminars/Conferences/Workshops on relevant topic.

2. Second year:

i) To attend the seminars/Conferences/Workshops on relevant topic.
ii) To formulate new thermoelastic problem on different solid.
iii) To study existing and new technique which will be applied on these Problems.
iv) To construct the new mathematical models for various solids and study their thermoelasticity.
v) To check whether these models are practically feasible or not which will be discussed with engineering researchers.

3. Third year:

i) To construct the new mathematical models for various solids and study their thermoelasticity.
ii) To study applications of obtained results.
iii) To publish the work in reputed journals.
iv) To submit the work to the authority.

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